SEÑALES Y SISTEMAS

Práctica 2
Ejercicio 2

Sebastián Pazos

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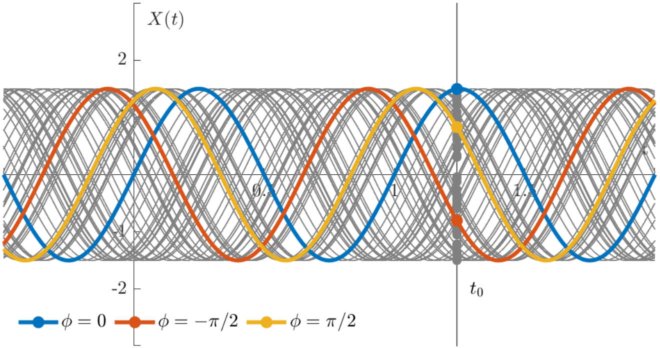
Repaso: Procesos Estocásticos. Ejemplo

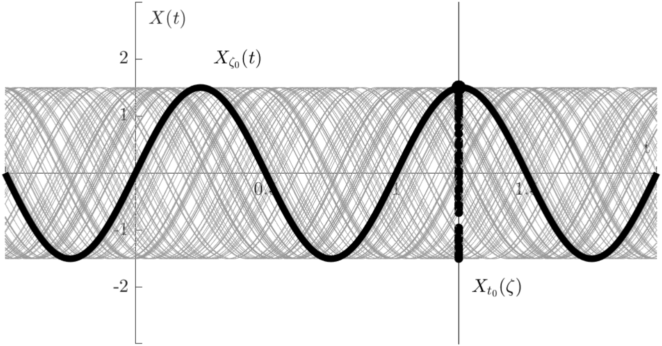
Generador sinusoidal, $X(t) = A\cos(\omega_0 t + \phi)$, ϕ: V.A. $\sim \mathcal{U}[-\pi, \pi]$



para cada valor de ϕ una **realización del proceso**
para cada valor de t una **variable aleatoria**

Conjunto de variables aleatorias ordenadas.

$$X(t,\zeta) : T \times \Omega \to \mathbb{R}$$



$\zeta = \zeta_0 \to X_{\zeta_o}(t)$ **REALIZACIÓN**

$t = t_0 \to X_{t_o}(\zeta)$ **VARIABLE ALEATORIA**

Distribuciones

$$\begin{aligned} F_X(x;t) &= P\{X_t \leq x\} = \mathcal{P}\{\zeta \in \Omega : X(t,\zeta) \leq x\} \\ &= P\{X(t) \leq x\} \\ f_X(x;t) &= \frac{\partial F_X(x;t)}{\partial x} \end{aligned}$$

Media

$$\begin{aligned} E\{X_t(\zeta)\} &= \int_{-\infty}^{\infty} x f_X(x;t)\, dx = \mu(t) \\ E\{X(t)\} &= \mu(t) \end{aligned}$$

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a.I. $X(t) = A$ donde A: V.A. con $f_A(\cdot)$

$$E\{X(t)\} = E\{A\} = \mu_A$$

$$f_X(x;t) = f_A(x)$$

a.II. $X(t) = A + t$ donde A: V.A. con $f_A(\cdot)$

$$E\{X(t)\} = E\{A + t\} = E\{A\} + E\{t\} = \mu_A + t$$

$$\begin{aligned} F_X(x;t) &= P\{X(t) \leq x\} = P\{A + t \leq x\} \\ &= P\{A \leq x - t\} \\ &= F_A(x - t) \end{aligned}$$

$$f_X(x;t) = f_A(x - t)$$

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a.III. $X(t) = Ae^{-t}$ donde A: V.A. con $f_A(\cdot)$

$$E\{X(t)\} = E\{Ae^{-t}\} = E\{A\}e^{-t} = \mu_A e^{-t}$$

$$\begin{aligned} F_X(x;t) &= P\{X(t) \leq x\} = P\{Ae^{-t} \leq x\} \\ &= P\{A \leq xe^{t}\} \\ &= F_A(xe^{t}) \end{aligned}$$

$$f_X(x;t) = f_A(xe^{t})e^{t}$$

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b.I. $X[n] = B[n] + A$ donde B[n] secuencia aleatoria i.i.d. $f_B(\cdot)$ independiente de A: V.A. con $f_A(\cdot)$

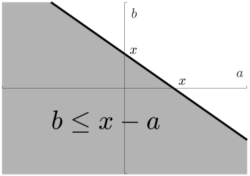
$$f_{AB}(a,b) = f_A(a) f_B(b)$$

$$E\{X[n]\} = E\{B[n] + A\} = E\{B_n\} + E\{A\} = \mu_B + \mu_A$$

$$F_X(x;n) = P\{X_n \leq x\} = P\{B_n + A \leq x\}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{x-a} f_{AB}(a,b)\, db\, da$$

$$f_X(x;n) = \int_{-\infty}^{\infty} f_{AB}(a, x-a)\, da$$



Como son independientes

$$f_X(x;n) = \int_{-\infty}^{\infty} f_A(a) f_B(x-a)\, da$$

$$= (f_A * f_B)(x)$$

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b.II. $X[n] = nB[n]$ donde B[n] secuencia aleatoria i.i.d. $f_B(\cdot)$

$$E\{X[n]\} = E\{nB_n\} = nE\{B_n\} = n\mu_B$$

$$\begin{aligned} F_X(x;n) &= P\{X_n \leq x\} = P\{nB_n \leq x\} \\ &= P\{B_n \leq x/n\} \\ &= F_B(x/n) \end{aligned}$$

$$f_X(x;n) = f_B(x/n)/n$$

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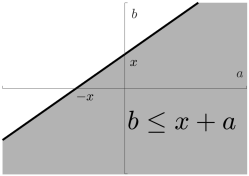
b.III. $X[n] = B[n] - B[n-1]$ donde $B[n]$ secuencia aleatoria i.i.d. $f_B(\cdot)$

$$\begin{aligned} E\{X[n]\} &= E\{B[n] - B[n-1]\} = E\{B_n\} - E\{B_{n-1}\} \\ &= \mu_B - \mu_B = 0 \end{aligned}$$

$$\begin{aligned} F_X(x;n) &= P\{X_n \leq x\} = P\{B_n - B_{n-1} \leq x\} \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{x+a} f_{B_n B_{n-1}}(b,a)\, db\, da \end{aligned}$$

$$f_X(x;n) = \int_{-\infty}^{\infty} f_{B_n B_{n-1}}(x+a,a)\, da$$



Como es i.i.d.

$$f_X(x;n) = \int_{-\infty}^{\infty} f_B(x+a) f_B(a)\, da = (f_B \star f_B)(x)$$

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d. Random walk

$$W[n] = \sum_{k=1}^{n} S[k] \qquad W[0] = 0$$

$$E\{W[n]\} = E\left\{\sum_{k=1}^{n} S[k]\right\} = \sum_{k=1}^{n} E\{S[k]\} = 1\frac{1}{2} + (-1)\frac{1}{2} = 0$$

$$Var\{W[n]\} = Var\left\{\sum_{k=1}^{n} S[k]\right\} = \sum_{k=1}^{n} Var\{S[k]\} = \sum_{k=1}^{n} 1 = n$$

$$Var\{S[k]\} = E\left\{S[k]^2\right\} = 1^2\frac{1}{2} + (-1)^2\frac{1}{2} = 1$$

$$S[k] = \left\{ \begin{array}{cc} 1 & 1/2 \\ -1 & 1/2 \end{array} \right. \longrightarrow f_S(s) = \frac{1}{2}\delta[s-1] + \frac{1}{2}\delta[s+1]$$

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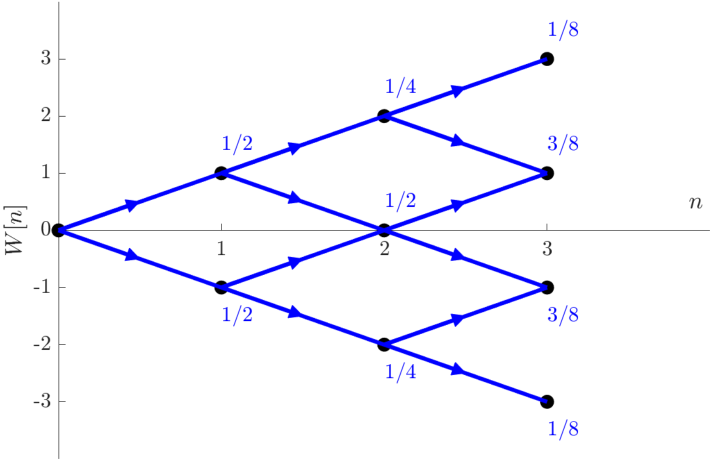
$f_W(w;n)$?



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$$W[1] = S[1] \longrightarrow f_W(w;1) = f_S$$
$$f_W(w;1) = \frac{1}{2}\delta[w-1] + \frac{1}{2}\delta[w+1]$$

$$W[2] = S[1] + S[2] \longrightarrow f_W(w;2) = (f_S * f_S)(w)$$
$$f_W(w;2) = \frac{1}{4}\delta[w-2] + \frac{1}{2}\delta[w] + \frac{1}{4}\delta[w+2]$$

$$W[3] = S[1] + S[2] + S[3] \longrightarrow f_W(w;3) = (f_W(\cdot;2) * f_S)(w)$$
$$f_W(w;3) = \frac{1}{8}\delta[w-3] + \frac{3}{8}\delta[w-1] + \frac{3}{8}\delta[w+1] + \frac{1}{8}\delta[w+3]$$