SEÑALES Y SISTEMAS

Práctica 2
Ejercicios 3 y 4

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Repaso: Correlación determinística

Correlación

$$r_{xy}(\tau) = \int_{-\infty}^{\infty} x(\tau + \lambda) y^*(\lambda) \, d\lambda$$

Autocorrelación

$$r_{xx}(\tau) = \int_{-\infty}^{\infty} x(\tau + \lambda) x^*(\lambda) \, d\lambda$$

$$r_{xx}(0) = \int_{-\infty}^{\infty} x(\lambda) x^*(\lambda) \, d\lambda = \int_{-\infty}^{\infty} |x(\lambda)|^2 \, d\lambda = E_x$$

Secuencias

$$r_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n + m] y^*[n]$$

Ejercicio 3

a.I. Simetría hermítica $r_{yx}^*(-\tau) = r_{xy}(\tau)$

$$\begin{aligned} r_{yx}^*(-\tau) &= \left(\int_{-\infty}^{\infty} y(-\tau+\lambda)x^*(\lambda)\,d\lambda\right)^* \\ &= \int_{-\infty}^{\infty} y^*(\lambda-\tau)x(\lambda)\,d\lambda \qquad u=\lambda-\tau, \qquad du = d\lambda \\ &= \int_{-\infty}^{\infty} x(\tau+u)y^*(u)\,du \\ &= r_{xy}(\tau) \end{aligned}$$

b.I. Señales de Potencia

$$r_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(\tau+\lambda)y^*(\lambda)\,d\lambda$$

$$r_{xx}(0) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(\lambda)x^*(\lambda)\,d\lambda = P_x \qquad r_{yy}(0) = P_y$$

Ejercicio 3

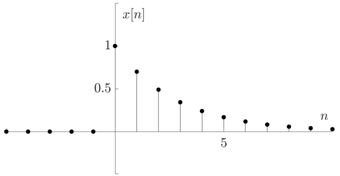
c.IV. $y[n] = ax[n] + bx[n-N]$

$$r_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n+m]\left(a^*x^*[n] + b^*x^*[n-N]\right)$$

$$= a\sum_{n=-\infty}^{\infty} x[n+m]x^*[n] + b\sum_{n=-\infty}^{\infty} x[n+m]x^*[n-N]$$

$$= ar_{xx}[m] + br_{xx}[m+N]$$

d.II. $x[n] = a^n u[n]$, $0 < a < 1$



$$r_{xx}[m] = \sum_{n=-\infty}^{\infty} x[n+m]x^*[n]$$

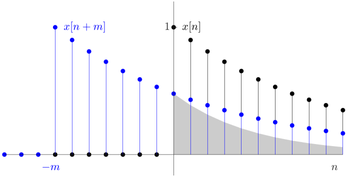
$$= \sum_{n=-\infty}^{\infty} a^{n+m}u[n+m]a^nu[n] = a^m \sum_{n=-\infty}^{\infty} a^{2n} \underbrace{u[n+m]}_{n\geq -m} \underbrace{u[n]}_{n\geq 0}$$

Ejercicio 3

si $m \geq 0$

$$r_{xx}[m] = a^m \sum_{n=0}^{\infty} \left(a^2\right)^n$$

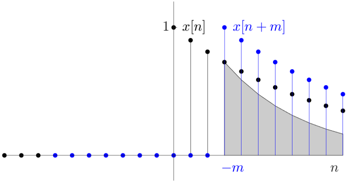
$$= a^m \frac{1}{1-a^2} = \frac{a^m}{1-a^2}$$



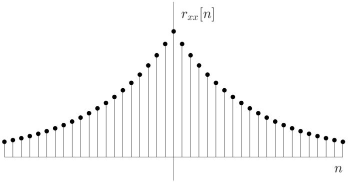
si $m < 0$

$$r_{xx}[m] = a^m \sum_{n=-m}^{\infty} \left(a^2\right)^n$$

$$= a^m \frac{\left(a^2\right)^{-m}}{1-a^2} = \frac{a^{-m}}{1-a^2}$$



$$r_{xx}[m] = \frac{a^{|m|}}{1-a^2}$$



Correlación

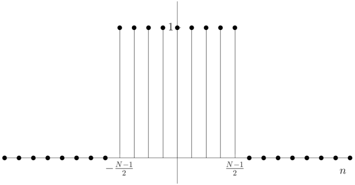
$$R_{XY}[n+m,n] = E\left\{X[n+m]Y^*[n]\right\}$$

Autocorrelación

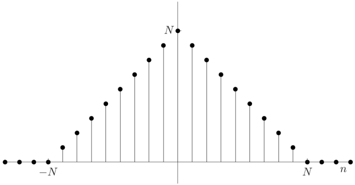
$$R_{XX}[n+m,n] = E\left\{X[n+m]X^*[n]\right\}$$

Recordar

$\sqcap_N[n]$



$\wedge_M[n]$



$$\left\{\sqcap_N[\cdot] * \sqcap_N[\cdot]\right\}[n] = \wedge_N[n]$$

Ejercicio 4

a. Hallar $R_{XX}[n+m,n]$ para $X[n] \sim \mathcal{U}[-1/2, 1/2]$ i.i.d.
si $m = 0$

$$R_{XX}[n,n] = E\left\{X[n]X^*[n]\right\} = E\left\{|X[n]|^2\right\} = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$= \int_{-1/2}^{1/2} x^2\,dx = \left.\frac{x^3}{3}\right|_{-1/2}^{1/2} = 1/24 + 1/24 = 1/12$$

si $m \neq 0$

$$R_{XX}[n+m,n] = E\left\{X[n+m]X^*[n]\right\}$$
$$= E\left\{X[n+m]\right\} E\left\{X^*[n]\right\} = 0$$

$$R_{XX}[m] = \frac{1}{12}\delta[m]$$

Ejercicio 4

c. $Y[n] = \sum_{i=0}^{11} X[n-i]$

$$R_{YY}[n+m,n] = E\left\{\sum_{i=0}^{11} X[n+m-i] \sum_{j=0}^{11} X^*[n-j]\right\}$$

$$= \sum_{i=0}^{11}\sum_{j=0}^{11} E\left\{X[n+m-i]X^*[n-j]\right\}$$

$$= \sum_{i=0}^{11}\sum_{j=0}^{11} R_{XX}[m-i+j]$$

$$= \sum_{i=0}^{11}\sum_{j=0}^{11} \frac{1}{12}\delta[m-i+j]$$

Ejercicio 4

$$
\begin{aligned}
R_{YY}[n+m,n] &= \frac{1}{12}\sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} \sqcap_{[0,11]}[i]\sqcap_{[0,11]}[j]\delta[m-i+j] \\
&= \frac{1}{12}\sum_{i=-\infty}^{\infty} \sqcap_{[0,11]}[i]\sqcap_{[0,11]}[i-m] \\
&= 1-|m|/12 \\
&= \frac{1}{12}\wedge_{12}[m]
\end{aligned}
$$