SEÑALES Y SISTEMAS

Práctica 3
Ejercicios 1, 2, 3 y 4

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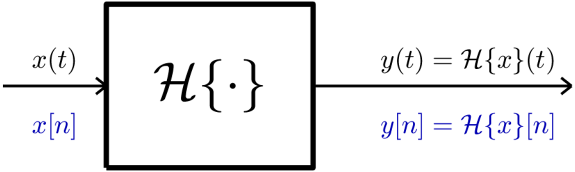
Sistemas. Propiedades



A **Con y sin memoria**. La salida depende/no depende de entradas anteriores.

2a **Ejemplo 1.** $y(t) = a\,x(t) + b$ SIN MEMORIA

2b **Ejemplo 2.** $y(t) = \int_{-\infty}^{t} x(\lambda)\,d\lambda$ CON MEMORIA

B Linealidad

- Homogeneidad: si $y(t) = \mathcal{H}\{x\}(t)$ entonces $\mathcal{H}\{c\,x\}(t) = c\,y(t)$
- Aditividad: si $y_1(t) = \mathcal{H}\{x_1\}(t)$ y $y_2(t) = \mathcal{H}\{x_2\}(t)$
  entonces $\mathcal{H}\{x_1 + x_2\}(t) = y_1(t) + y_2(t)$

2a **Ejemplo 1**. $y(t) = a\,x(t) + b$

Si $y_1(t) = a\,x_1(t) + b$ y $y_2(t) = a\,x_2(t) + b$

para $x_L(t) = \alpha\,x_1(t) + \beta\,x_2(t)$ tenemos

$$\begin{aligned} y_L(t) = a\,x_L(t) + b &= a(\alpha\,x_1(t) + \beta\,x_2(t)) + b \\ &= a\,\alpha\,x_1(t) + a\,\beta\,x_2(t) + b \end{aligned}$$

pero

$$\begin{aligned} \alpha\,y_1(t) + \beta\,y_2(t) &= \alpha(a\,x_1(t) + b) + \beta(a\,x_2(t) + b) \\ &= \alpha\,a\,x_1(t) + \beta\,a\,x_2(t) + \alpha\,b + \beta\,b \end{aligned}$$

$$y_L(t) \neq \alpha\,y_1(t) + \beta\,y_2(t)$$ NO LINEAL

2b **Ejemplo 2**. $y(t) = \int_{-\infty}^{t} x(\lambda)\, d\lambda$

si $\quad y_1(t) = \int_{-\infty}^{t} x_1(\lambda)\, d\lambda \quad$ y $\quad y_2(t) = \int_{-\infty}^{t} x_2(\lambda)\, d\lambda$

para $x_L(t) = \alpha\, x_1(t) + \beta\, x_2(t)$ tenemos

$$y_L(t) = \int_{-\infty}^{t} \alpha\, x_1(\lambda) + \beta\, x_2(\lambda)\, d\lambda$$

$$= \alpha \underbrace{\int_{-\infty}^{t} x_1(\lambda)\, d\lambda}_{y_1(t)} + \beta \underbrace{\int_{-\infty}^{t} x_2(\lambda)\, d\lambda}_{y_2(t)}$$

$$y_L(t) = \alpha\, y_1(t) + \beta\, y_2(t)$$ LINEAL

C **Invarianza en el tiempo** si $y(t) = \mathcal{H}\{x\}(t)$ entonces para $x_1(t) = x(t-\tau)$ la salida es $y_1(t) = y(t-\tau)$

B + C $\longrightarrow$ **SLIT**

2b **Ejemplo 1**. $y(t) = \int_{-\infty}^{t} x(\lambda)\, d\lambda$

si $x_1(\alpha) = x(\alpha - \tau)$ entonces $y_1(t) = \int_{-\infty}^{t} x(\lambda - \tau)\, d\lambda$

por otro lado

$$y(t-\tau) = \int_{-\infty}^{t-\tau} x(\lambda)\, d\lambda \qquad \lambda = \alpha - \tau \to \alpha = \lambda + \tau$$

$$= \int_{-\infty}^{t} x(\alpha - \tau)\, d\alpha$$

$y(t-\tau) = y_1(t)$ INVARIANTE AL DESPLAZAMIENTO

2c **Ejemplo 2.** $\dfrac{dy}{dt} = t^3 x(t)$

es equivalente a $y(t) = \int_{-\infty}^{t} \lambda^3 x(\lambda)\, d\lambda$

si $x_1(\alpha) = x(\alpha - \tau)$ entonces $y_1(t) = \int_{-\infty}^{t} \lambda^3 x(\lambda - \tau)\, d\lambda$

por otro lado

$$y(t-\tau) = \int_{-\infty}^{t-\tau} \lambda^3 x(\lambda)\, d\lambda \qquad \lambda = \alpha - \tau \rightarrow \alpha = \lambda + \tau$$

$$= \int_{-\infty}^{t} (\alpha - \tau)^3 x(\alpha - \tau)\, d\alpha$$

$y(t-\tau) \neq y_1(t)$ VARIANTE AL DESPLAZAMIENTO

D **Invarianza al desplazamiento** si $y[n] = \mathcal{H}x[n]$ entonces para $x_1[n] = x[n-m]$ la salida es $y_1[n] = y[n-m]$

B + D $\longrightarrow$ **SLID**

1a **Ejemplo 3**. para $x[n] \longrightarrow y[n] = x[n^2]$

si $x_1[m] = x[m-N]$ entonces $y_1[n] = x_1[n^2] = x[n^2 - N]$

pero $y[n-N] = x\left[(n-N)^2\right] = x\left[n^2 - 2nN + N^2\right]$

$y[n-N] \neq y_1[n]$ VARIANTE AL DESPLAZAMIENTO

E **Causalidad**

Cambio en entrada	$\longrightarrow$	Respuesta al cambio en la salida **posterior**

2f **Ejemplo**. $y[n] = x[-n]$

$$y[1] = x[-1]$$
$$y[0] = x[0]$$
$$y[-1] = x[1]$$

el pasado depende del futuro NO CAUSAL

F **Estabilidad EASA (BIBO)** si $|x(t)| < M$ entonces la salida $|y(t)| < N$

2b **Ejemplo** $y(t) = \int_{-\infty}^{t} x(\lambda)\, d\lambda$

$$|y(t)| = \left| \int_{-\infty}^{t} x(\tau)\, d\tau \right| \leq \int_{-\infty}^{t} |x(\tau)|\, d\tau \leq M \int_{-\infty}^{t} d\tau = \infty$$

Contraejemplo:

$x(t) = 1$ entrada acotada pero

$|y(t)| = \infty$ salida no acotada

NO ESTABLE

Ejercicio 3

b. Cascada de NL = NL **FALSO** CONTRAEJEMPLO

$$\mathcal{S}_1 : x_1[n] \longrightarrow y_1[n] = (x_1[n])^3$$

$$\mathcal{S}_2 : x_2[n] \longrightarrow y_2[n] = \sqrt[3]{x_2[n]}$$

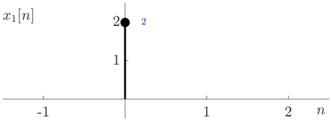
$$\mathcal{S}_1\mathcal{S}_2 : x_1[n] \longrightarrow y_{12}[n] = x_1[n]$$ LINEAL

c. Cascada de ID = ID **VERDADERO**

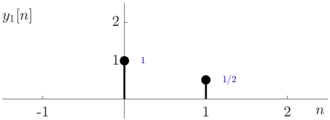
$$x_1[n] \xrightarrow{\mathcal{S}_1} y_1[n] \xrightarrow{\mathcal{S}_2} y_2[n]$$

$$x_1[n-m] \xrightarrow{\text{ID}} y_1[n-m] \xrightarrow{\text{ID}} y_2[n-m]$$

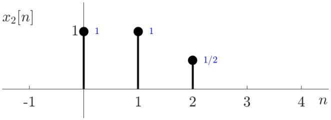
a.



$\longrightarrow$



$$x_1[n] = 2\delta[n]$$



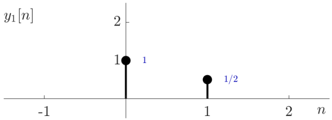
$$x_2[n] = \delta[n] + \delta[n-1] + \frac{1}{2}\delta[n-2]$$

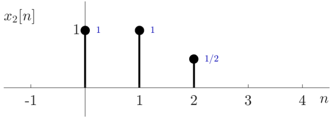
$$x_2[n] = \frac{1}{2}x_1[n] + \frac{1}{2}x_1[n-1] + \frac{1}{4}x_1[n-2]$$ SLID

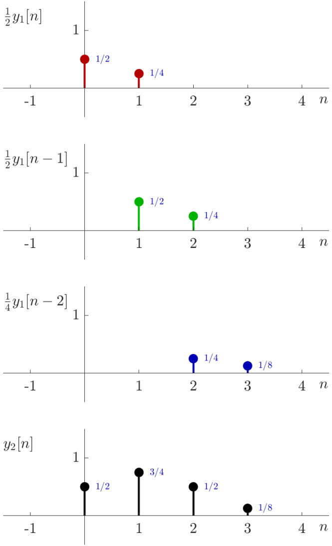
$$y_2[n] = \frac{1}{2}y_1[n] + \frac{1}{2}y_1[n-1] + \frac{1}{4}y_1[n-2]$$

Ejercicio 4

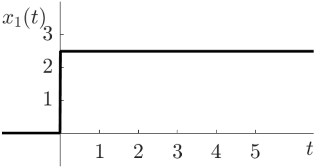
$$y_2[n] = \frac{1}{2}y_1[n] + \frac{1}{2}y_1[n-1] + \frac{1}{4}y_1[n-2]$$



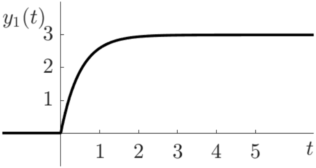




b.

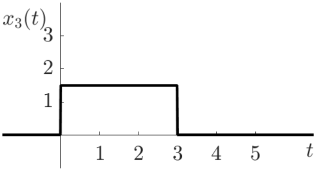


$\longrightarrow$



$$x_1(t) = \tfrac{5}{2}u(t)$$

$$y_1(t) = 3\left(1 - e^{-2t}\right)u(t)$$



$$x_3(t) = \frac{3}{2}u(t) - \frac{3}{2}u(t-3)$$

$$= \frac{5}{3}x_1(t) - \frac{5}{3}x_1(t-3)$$

$$y_3(t) = \ldots$$ SLIT

c. $x_I(t) = \delta(t)$

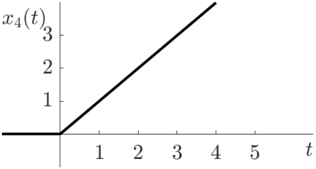
$$\frac{d}{dt}\left\{u\right\}(t) = \delta(t) \quad \longrightarrow \quad x_I(t) = \frac{2}{5}\frac{d}{dt}\left\{x_1\right\}(t)$$

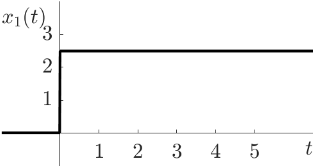
$$\begin{aligned} y_I(t) = \mathcal{H}\{x_I\}(t) &= \frac{2}{5}\mathcal{H}\left\{\frac{d}{dt}\left\{x_1\right\}\right\}(t) \\ &= \frac{2}{5}\frac{d}{dt}\left\{\mathcal{H}\{x_1\}\right\}(t) \\ &= \frac{2}{5}\frac{d}{dt}\left\{y_1\right\}(t) \\ &= \frac{2}{5}\frac{d}{dt}\left\{\left(3 - 3e^{-2t}\right)u(t)\right\} \\ &= \frac{2}{5}(\underbrace{\left(3 - 3e^{-2t}\right)\delta(t)}_{0} + 6e^{-2t}u(t)) = \frac{12}{5}e^{-2t}u(t) \end{aligned}$$

SL CONMUTATIVOS

DERIVADA PRODUCTO

d. $x_4(t) = t\,u(t)$





$$x_4(t) = \int_0^t \frac{2}{5} x_1(\lambda)\,d\lambda \qquad \forall t \geq 0$$

$$\begin{aligned} y_4(t) &= \mathcal{H}\{x_4\}(t) \\ &= \mathcal{H}\left\{\int_0^t \frac{2}{5} x_1(\lambda)\,d\lambda\right\}(t) \\ &= \int_0^t \frac{2}{5} \mathcal{H}\{x_1\}(\lambda)\,d\lambda \\ &= \frac{2}{5}\int_0^t y_1(\lambda)\,d\lambda \\ &= \frac{2}{5}\int_0^t 3 - 3e^{-2\lambda}\,d\lambda = \left(\frac{6}{5}t - \frac{1-e^{-2t}}{2}\,d\lambda\right) u(t) \end{aligned}$$