SEÑALES Y SISTEMAS

Práctica 3
Ejercicios 5 y 6

Sebastián Pazos

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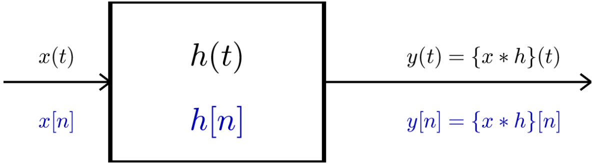
Respuesta impulsional



Causalidad

$$h(t) = 0 \qquad \forall \quad t < 0 \qquad \text{ó}$$
$$h[n] = 0 \qquad \forall \quad n < 0$$

Estabilidad

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty \longrightarrow \text{ESTABLE}$$

$$\sum_{n=\infty}^{\infty} |h[n]| < \infty \longrightarrow \text{ESTABLE}$$

Convolución

SLIT

$$y(t) = \{x * h\}(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\, d\tau = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\, d\lambda$$

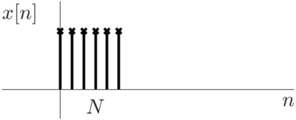
SLID

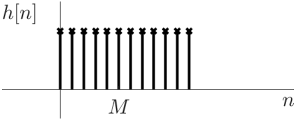
$$y[n] = \{x * h\}[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{m=-\infty}^{\infty} x[n-m]h[m]$$

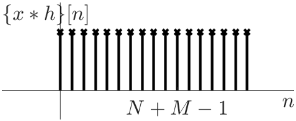
Soporte de x
$\boldsymbol{N}$

Soporte de h
$\boldsymbol{M}$

Soporte de $x * h$
$\boldsymbol{N + M - 1}$



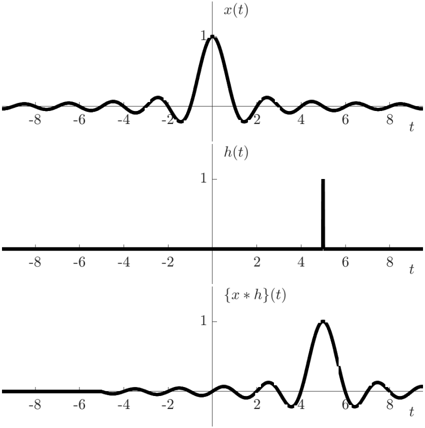




Ejercicio 5

a.I $x(t) \qquad h(t) = \delta(t-5)$

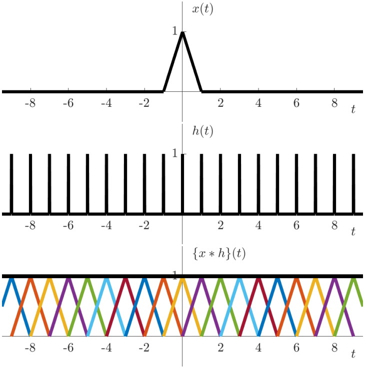
$$\begin{aligned} y(t) &= \{x * h\}(t) \\ &= \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau \\ &= \int_{-\infty}^{\infty} x(t-\tau)\delta(\tau-5)\,d\tau \\ &= x(t-5) \end{aligned}$$



Ejercicio 5

Ejemplo interesante $x(t) = \bigwedge(t)$ $h(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$

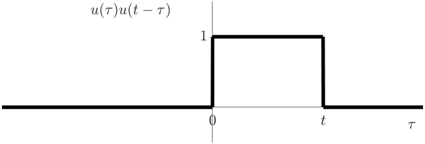
$$
\begin{aligned}
y(t) &= \{x * h\}(t) \\
&= \int_{-\infty}^{\infty} \bigwedge(t-\tau) \sum_{n=-\infty}^{\infty} \delta(\tau - n)\, d\tau \\
&= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \bigwedge(t-\tau)\delta(\tau - n)\, d\tau \\
&= \sum_{n=-\infty}^{\infty} \bigwedge(t-n) \\
&= 1
\end{aligned}
$$



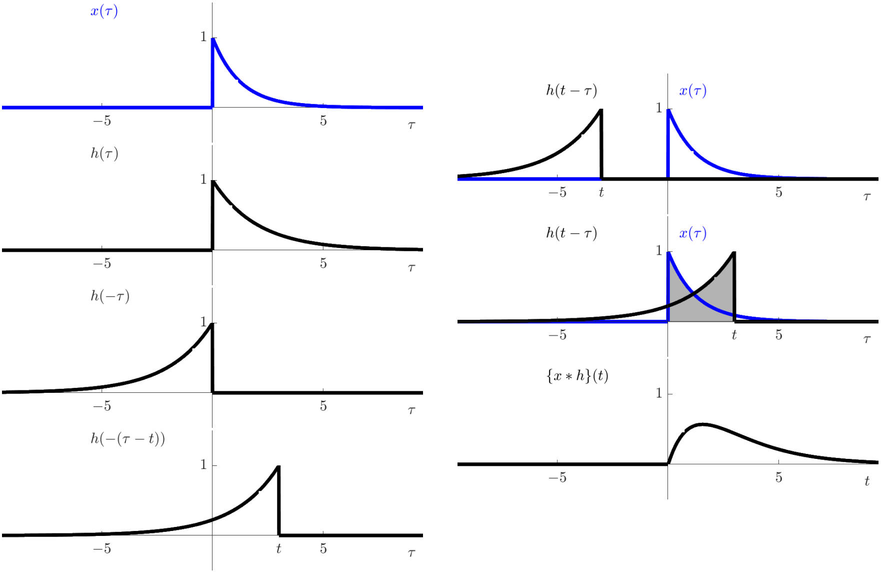
Ejercicio 5

a.III $x(t) = e^{-\alpha t}u(t) \qquad h(t) = e^{-\beta t}u(t)$

$$
\begin{aligned}
y(t) &= \{x * h\}(t) \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau \\
&= \int_{-\infty}^{\infty} e^{-\alpha\tau}u(\tau)e^{-\beta(t-\tau)}u(t-\tau)\, d\tau \\
&= \int_{0}^{t} e^{-\alpha\tau}e^{-\beta t}e^{\beta\tau}\, d\tau\, u(t) \\
&= e^{-\beta t}\int_{0}^{t} e^{-\alpha\tau}e^{\beta\tau}\, d\tau\, u(t) \\
&= \begin{cases} \frac{e^{-\alpha t}-e^{-\beta t}}{\beta-\alpha}u(t) & \alpha \neq \beta \\ e^{-\alpha t}tu(t) & \alpha = \beta \end{cases}
\end{aligned}
$$

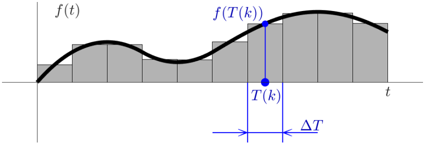


Ejercicio 5


x(τ)
1
−5
5
τ
h(τ)
1
−5
5
τ
h(−τ)
1
−5
5
τ
h(−(τ − t))
1
−5
t
5
τ
h(t − τ)
x(τ)
1
−5
t
5
τ
h(t − τ)
x(τ)
1
−5
t
5
τ
{x ∗ h}(t)
1
−5
5
t

Convolución continua en Matlab

$$\int_{-\infty}^{\infty} f(t)\,dt \approx \sum_{k=-N}^{N} f(T(k))\Delta T$$



$$\begin{aligned}\{x * h\}(t) &= \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau \\ &\approx \sum_{k=-N}^{N} x(T(n)-T(k))h(T(k))\Delta T \\ &= \Delta T \sum_{k=-N}^{N} \widetilde{x}[n-k]\widetilde{h}[k] \\ &= \{\widetilde{x} * \widetilde{h}\}[n]\Delta T\end{aligned}$$

Ejemplo Ej. 4

$$x_1(t) = \frac{5}{2}u(t) \longrightarrow y_1(t) = 3\left(1 - e^{-2t}\right)u(t)$$

$$h(t) = \frac{12}{5}e^{-2t}u(t)$$

```
dt = 0.001;                                       % Paso de tiempo para aproximar 'lo continuo'
t = -2:dt:10;                                     % Vector de tiempo
t_conv = -4:dt:20;                                % Vector de tiempo de la convolucion. El soporte
                                                  % de la convolución es la suma de los soportes
                                                  % cada señal a convolucionar.

h = 12/5.*exp(-2.*t).*(t >= 0);                   % Rta. impulsional sistema continuo. Ej 4.
x1 = 5/2.*(t >= 0);                               % Primera entrada x1(t)

y1 = conv(x1,h)*dt;                               % No olvidar multiplicar por el paso dt.
y1_teo = 3.*(1-exp(-2.*t_conv)).*(t_conv >= 0); % Cálculo teórico del Ej 4.

plot(t,h2);                                       % Señal
plot(t,x1);                                       % Sistema
plot(t_conv,y1);                                  % Convolucion
```