SEÑALES Y SISTEMAS

Práctica 3
Ejercicios 7 y 8

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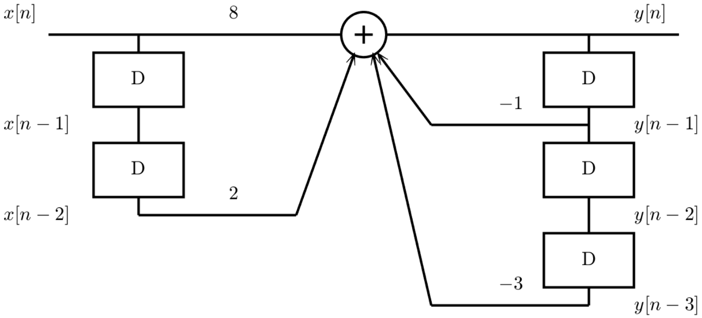
20 de abril de 2020

$$\mathcal{S} : y[n] = 8x[n] + 2x[n-2] - y[n-1] - 3y[n-3]$$

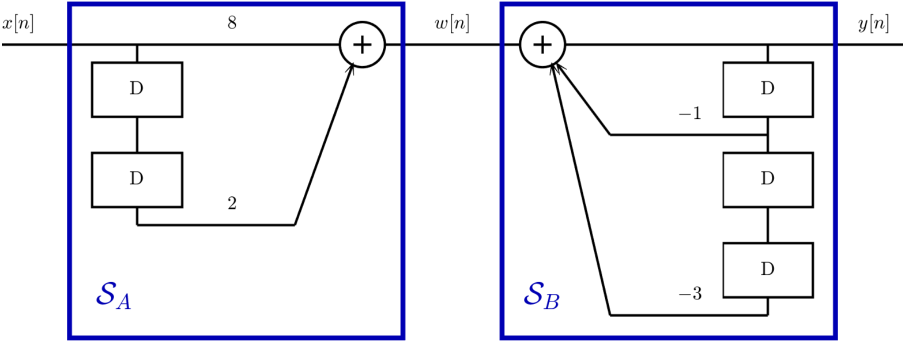
Forma Directa I



Necesitamos $x[n-1]$, $x[n-2]$, $y[n-1]$, $y[n-2]$ y $y[n-3]$

$$\mathcal{S} : y[n] = 8x[n] + 2x[n-2] - y[n-1] - 3y[n-3]$$

Forma Directa II



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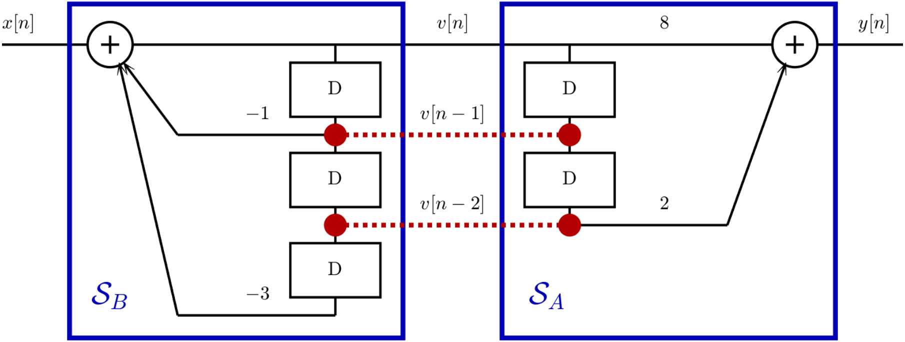
Forma Directa II



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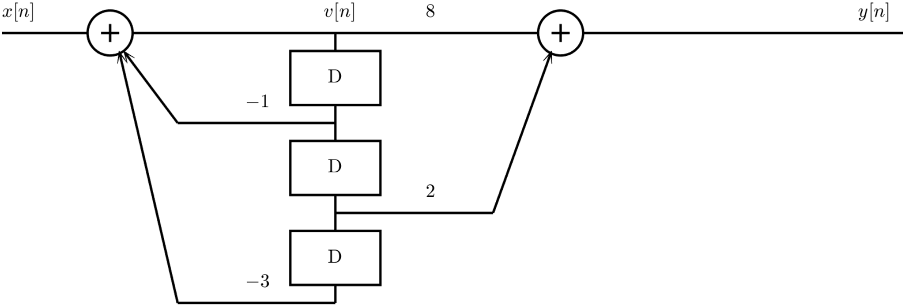
Forma Directa II



$$\mathcal{S}: v[n] = x[n] - v[n-1] - 3v[n-3] \qquad y[n] = 8v[n] + 2v[n-2]$$

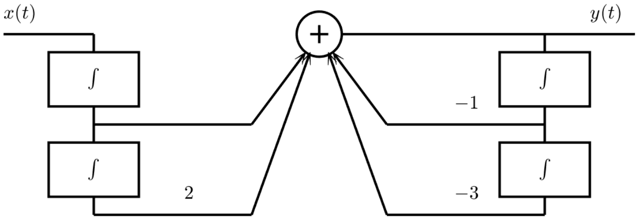
Necesitamos $v[n-1]$, $v[n-2]$ y $v[n-3]$

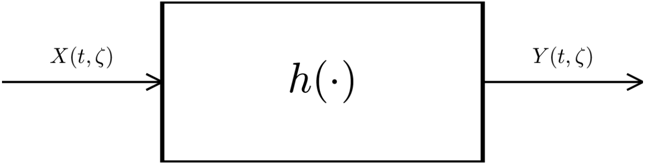
Ejercicio 7. Ecuaciones Diferenciales

$$\mathcal{S} : \ddot{y}(t) + \dot{y}(t) + 3y(t) = 2x(t) + \dot{x}(t)$$

$$\iint \ddot{y}(t)\,dt\,dt + \iint \dot{y}(t)\,dt\,dt + \iint 3y(t)\,dt\,dt = \iint 2x(t)\,dt\,dt + \iint \dot{x}(t)\,dt\,dt$$

$$y(t) + \int y(t)\,dt + \iint 3y(t)\,dt\,dt = \iint 2x(t)\,dt\,dt + \int x(t)\,dt$$





$$Y(t,\zeta) = \int_{-\infty}^{\infty} X(\tau,\zeta)\bar{h}(t,\tau)\,d\tau$$

Media

$$E\{Y(t)\} = \{\mu_X * h\}(t) = \int_{-\infty}^{\infty} \mu_X(\tau)h(t-\tau)\,d\tau$$

- Si h es estable, $\mu_x(t)$ es constante y la entrada se aplica en $t=-\infty$ entonces $\mu_Y(t)$ también es constante
- Si la entrada se aplica en $t=0$ entonces $\mu_Y(t)$ depende de t TRANSITORIO

SLIT con entrada ESA en $t=-\infty$, con $\check{h}(t)=h^*(-t)$

Intercorrelación

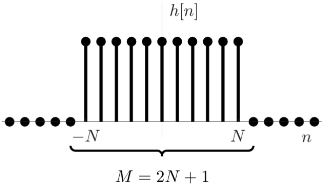
$$R_{XY}(t_1,t_2)=E\{X(t_1)Y^*(t_2)\}=\{\check{h}*R_X\}(t_1-t_2)$$
$$R_{YX}(t_1,t_2)=E\{Y(t_1)X^*(t_2)\}=\{h*R_X\}(t_1-t_2)$$

Autocorrelación

$$R_{YY}(t_1,t_2)=E\{Y(t_1)Y^*(t_2)\}=\{h*\check{h}*R_X\}(t_1-t_2)$$

la salida es ESA

$$h[n] = \frac{1}{2N+1} \sum_{m=-N}^{N} \delta[n-m]$$



$h[n]$ es NO CAUSAL

$$Y[n] = \frac{1}{2N+1} \sum_{m=-N}^{N} X[n-m]$$

Ayuditas

$$\sum_{n=A}^{B} r^n = \frac{r^A - r^{B+1}}{1-r}$$

$$e^{ja} - e^{jb} = e^{j\frac{a+b}{2}} \left(e^{j\frac{a-b}{2}} - e^{-j\frac{a-b}{2}} \right) = e^{j\frac{a+b}{2}} 2j \operatorname{sen}\left(\frac{a-b}{2}\right)$$

c.

$$\sum_{m=-N}^{N} \operatorname{sen}\left(\frac{2\pi}{M}(n-m)\right) = \frac{1}{2j}\sum_{m=-N}^{N} e^{j\frac{2\pi}{M}n}e^{-j\frac{2\pi}{M}m} - e^{-j\frac{2\pi}{M}n}e^{j\frac{2\pi}{M}m}$$

$$= \frac{e^{j\frac{2\pi n}{M}}}{2j}\left(\frac{e^{j\frac{2\pi N}{M}} - e^{-j\frac{2\pi(N+1)}{M}}}{1-e^{-j\frac{2\pi}{M}}}\right) - \frac{e^{-j\frac{2\pi n}{M}}}{2j}\left(\frac{e^{-j\frac{2\pi N}{M}} - e^{j\frac{2\pi(N+1)}{M}}}{1-e^{j\frac{2\pi}{M}}}\right)$$

$$= \frac{e^{j\frac{2\pi n}{M}}}{2j}\frac{e^{-j\frac{2\pi}{M}\frac{1}{2}}}{e^{-j\frac{2\pi}{M}\frac{1}{2}}}\left(\frac{e^{j\frac{2\pi(N+1/2)}{M}} - e^{-j\frac{2\pi(N+1/2)}{M}}}{e^{j\frac{\pi}{M}} - e^{-j\frac{\pi}{M}}}\right) + \cdots$$

$$= \frac{e^{j\frac{2\pi n}{M}}}{2j}\frac{\operatorname{sen}\left(\frac{\pi(2N+1)}{M}\right)}{\operatorname{sen}\left(\frac{\pi}{M}\right)} - \frac{e^{-j\frac{2\pi n}{M}}}{2j}\frac{\operatorname{sen}\left(\frac{\pi(2N+1)}{M}\right)}{\operatorname{sen}\left(\frac{\pi}{M}\right)} \quad M = 2N+1$$

$$= \frac{\operatorname{sen}\left(\frac{2\pi n}{M}\right)\operatorname{sen}(\pi)}{\operatorname{sen}\left(\frac{\pi}{M}\right)}$$

$$= 0$$

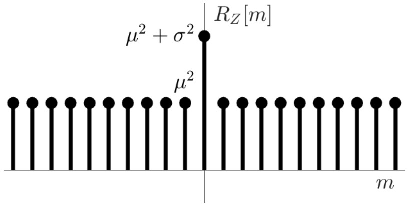
Ejercicio 8

d.I **Secuencia i.i.d.** $Z[n]$ i.i.d con $E\{Z[n]\} = \mu$ y $\mathrm{Var}\{Z[n]\} = \sigma^2$

$$R_{ZZ}[n+m,n] = E\{Z[n+m]Z[n]\}$$

$$= \begin{cases} E\{Z[n]Z[n]\} = \sigma^2 + \mu^2 & m = 0 \\ E\{Z[n+m]\}E\{Z[n]\} = \mu^2 & m \neq 0 \end{cases}$$

$$= \mu^2 + \sigma^2\delta[m]$$



d.III. $X[n] = A + V[n]$, $V[n] \sim \mathcal{N}(0, \sigma^2)$ i.i.d. $\rightarrow R_V[m] = \sigma^2 \delta[m]$

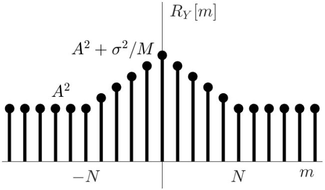
$$
\begin{aligned}
R_{YY}[n+m, n] &= E\{Y[n+m]Y[n]\} \\
&= E\left\{\left(A + \frac{1}{M}\sum_{k=-N}^{N} V[n+m-k]\right)\left(A + \frac{1}{M}\sum_{l=-N}^{N} V[n-l]\right)\right\} \\
&= A^2 + \frac{1}{M^2}\sum_{k=-N}^{N}\sum_{l=-N}^{N} E\{V[n+m-k]V[n-l]\} \\
&= A^2 + \frac{1}{M^2}\sum_{k=-N}^{N}\sum_{l=-N}^{N} R_V[n+m-k, n-l] \\
&= A^2 + \frac{1}{M^2}\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} \sigma^2 \delta[m-k+l] \sqcap_M[k] \sqcap_M[l]
\end{aligned}
$$

$$R_{YY}[n+m,n] = A^2 + \frac{1}{M^2}\sum_{k=-\infty}^{\infty} \sigma^2 \sqcap_M[k] \sum_{l=-\infty}^{\infty} \delta[m-k+l] \sqcap_M[l]$$

$$= A^2 + \frac{\sigma^2}{M^2}\sum_{k=-\infty}^{\infty} \sqcap_M[k] \sqcap_M[k-m]$$

$$= A^2 + \frac{\sigma^2}{M^2}\{\sqcap_M * \sqcap_M\}[m]$$

$$= A^2 + \frac{\sigma^2}{M^2} \wedge_M[m] \qquad \text{ESA}$$



d.IV.

$$V[n] \text{ Gaussiana} \rightarrow X[n] = A + V[n] \text{ Gaussiana}$$

$$\rightarrow Y[n] = \frac{1}{2N+1}\sum_{m=-\infty}^{\infty} X[n-m] \text{ Gaussiana}$$

Gaussiano + ESA → ESE

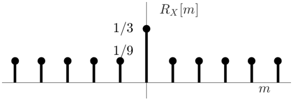
Ejercicio 8

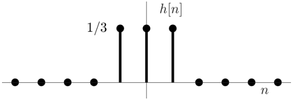
Convolución con constante

$$\{x * A\}[n] = \sum_{m=-\infty}^{\infty} x[m]A = A \sum_{m=-\infty}^{\infty} x[m] = \text{CTE}$$

f.II

$$\left.\begin{aligned} E\{X[n]\} &= \frac{1}{3} \\ R_X[m] &= \frac{2}{9}\delta[m] + \frac{1}{9} \end{aligned}\right\} X[n] \text{ ESA}$$



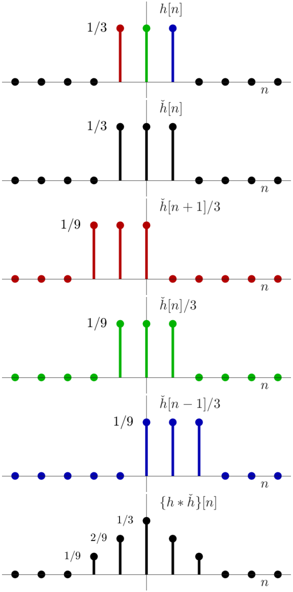


$$h[n] = \frac{1}{3} \sqcap_3[n]$$

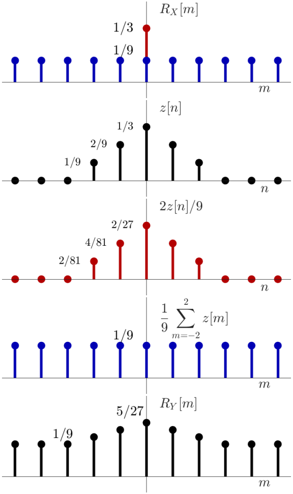
$$Y[n] = \frac{1}{3}(X[n-1] + X[n] + X[n+1])$$

$$E\{Y[n]\}[m] = \frac{1}{3}\left(E\{X[n-1]\} + E\{X[n]\} + E\{X[n+1]\}\right) = \frac{1}{3}$$

$$R_Y[m] = \{R_X * h * \check{h}\}[m]$$
$$= \{R_X * (h * \check{h})\}[m]$$
$$z[m] = \{h * \check{h}\}[m]$$
$$= \left\{\frac{1}{3}\sqcap_3 \frac{1}{3} * \sqcap_3\right\}[m]$$
$$= \frac{1}{9}\bigwedge{}_3[m]$$



$$
\begin{aligned}
R_Y[m] &= \{R_X * z\}[m] \\
&= \left\{\left(\frac{2}{9}\delta[\cdot] + \frac{1}{9}\right) * z\right\}[m] \\
&= \left\{\frac{2}{9}\delta[\cdot] * z\right\}[m] + \left\{\frac{1}{9} * z\right\}[m] \\
&= \frac{2}{9}z[m] + \frac{1}{9}\sum_{m=-\infty}^{\infty} z[m] \\
&= \frac{2}{81}\wedge_3[m] + \frac{1}{9}
\end{aligned}
$$



f.II

$Y[n]$ es ESA.
$Y[n]$ NO es i.i.d.