# SEÑALES Y SISTEMAS

## Práctica 4
## Ejercicios 1 y 2

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# Transformada de Fourier. Propiedades

$$X(f) = \mathcal{F}\{x\} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt$$
$$x(t) = \mathcal{F}^{-1}\{X\} = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,df$$

**Propiedades** $x(t) \supset X(f)$

- Simetría

$$\begin{array}{cccccccc} x = & p_R & +j & p_I & + & n_R & +j & n_I \\ & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ X = & P_R & +j & P_I & +j & N_I & + & N_R \end{array}$$

$x \in \mathbb{R} \Rightarrow X(f) = X^*(-f)$ Simetría hermítica

# Transformada de Fourier. Propiedades

- Dualidad

$$X(-t) \supset x(f)$$

- Linealidad $\quad y(t) \supset Y(f)$

$$\mathcal{F}\{x + y\} = X(f) + Y(f)$$

- Áreas

$$X(0) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft}\, dt = \text{ Área debajo de } x(t)$$

$$x(0) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft}\, df = \text{ Área debajo de } X(f)$$

# Transformada de Fourier. Propiedades

- Traslación

$$x(t - t_0) \supset e^{-j2\pi f t_0} X(f)$$
$$x(t)e^{j2\pi f_0 t} \supset X(f - f_0)$$

- Similaridad

$$x(a\,t) \supset \frac{1}{|a|} X\left(\frac{f}{a}\right)$$
$$x(-t) \supset X\left(-f\right)$$

- Derivación

$$\frac{dx}{dt} \supset j2\pi f X(f)$$
$$-j2\pi t x(t) \supset \frac{dX}{df}$$

- Integración

$$\int_{-\infty}^{t} x(\lambda)\, d\lambda \supset \frac{X(f)}{j2\pi f} + \frac{X(0)}{2}\delta(t)$$

- Convolución

$$\{x * y\}(t) \supset X(f)Y(f)$$
$$x(t)y(t) \supset \{X * Y\}(f)$$

**Igualdad en distribución**

$$\delta(t) \supset 1$$
$$1 \supset \delta(f)$$
$$\mathcal{F}\{1\} = \int_{-\infty}^{\infty} e^{j2\pi f t}\, df \equiv \delta(t)$$

$$X(-t) \supset x(f)$$

**Transformada de** $\sqcap(t)$

$$\begin{aligned}\mathcal{F}\{\sqcap\}(f) &= \int_{-\infty}^{\infty} \sqcap(t) e^{-j2\pi f t}\, dt \\ &= \int_{-1/2}^{1/2} e^{-j2\pi f t}\, dt \\ &= \begin{cases} 1 & f = 0 \\ \dfrac{e^{-j\pi f} - e^{j\pi f}}{-j2\pi f} = \dfrac{\operatorname{sen}(\pi f)}{\pi f} & f \neq 0 \end{cases} \\ &= \operatorname{sinc}(f)\end{aligned}$$

**Transformada de** $\wedge(t)$

$$\wedge(t) = \{\sqcap * \sqcap\}(t)$$

$$\wedge(t) \supset \operatorname{sinc}^2(f)$$

$$\{x * y\}(t) \supset X(f)Y(f)$$

# Transformada de Fourier. Pares

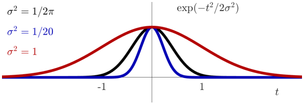
$$\delta(t) \supset 1 \qquad e^{-\alpha t}u(t) \supset \frac{1}{\alpha + j2\pi f}$$

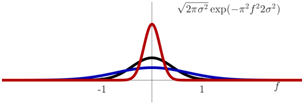
$$1 \supset \delta(f) \qquad \sqcap(t) \supset \mathrm{sinc}(f)$$

$$e^{j2\pi f_0 t} \supset \delta(f - f_0) \qquad \wedge(t) \supset \mathrm{sinc}^2(f)$$

$$\cos(2\pi f_0 t) \supset \frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0) \qquad \mathrm{sgn}(t) \supset \frac{-j}{\pi f}$$

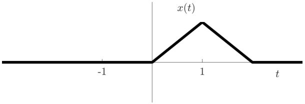
$$e^{-t^2/2\sigma^2} \supset \sqrt{2\pi\sigma^2}e^{-(\pi f)^2 2\sigma^2} \qquad u(t) \supset \frac{1}{2}\left(\delta(f) + \frac{1}{j\pi f}\right)$$



# Ejercicio 1

b $x(t) = \bigwedge(t-1)$

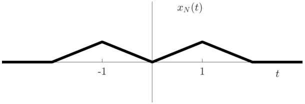



$$\bigwedge(t) \supset \text{sinc}^2(f)$$

$$\bigwedge(t-1) \supset \text{sinc}^2(f)e^{-j2\pi f}$$

$$x(t-t_0) \supset e^{-j2\pi f t_0} X(f)$$

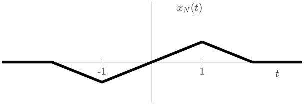
$$x_P(t) = \frac{\bigwedge(t-1) + \bigwedge(-t-1)}{2} = \frac{\bigwedge(t-1) + \bigwedge(t+1)}{2}$$




$$X_P(f) = \frac{\text{sinc}^2(f)e^{-j2\pi f} + \text{sinc}^2(f)e^{j2\pi f}}{2} = \text{sinc}^2(f)\cos(2\pi f)$$

# Ejercicio 1

$$x_N(t) = \frac{\bigwedge(t-1) - \bigwedge(t+1)}{2}$$




$$X_N(f) = \frac{\operatorname{sinc}^2(f)e^{-j2\pi f} - \operatorname{sinc}^2(f)e^{j2\pi f}}{2}$$
$$= -j\operatorname{sinc}^2(f)\operatorname{sen}(2\pi f)$$

$$X_P(f) + X_N(f) = \operatorname{sinc}^2(f)\left(\cos(2\pi f) - j\operatorname{sen}(2\pi f)\right)$$
$$= \operatorname{sinc}^2(f)\left(\cos(-2\pi f) + j\operatorname{sen}(-2\pi f)\right)$$
$$= \operatorname{sinc}^2(f)e^{-j2\pi f}$$

# Ejercicio 1

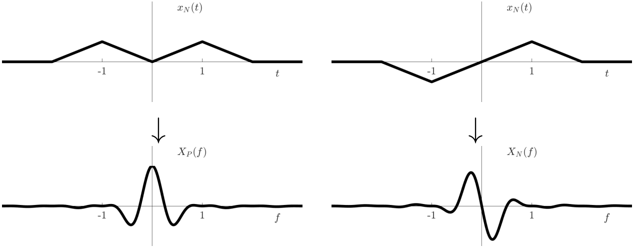
c $\quad x \in \mathbb{R} \Rightarrow X(f) = X^*(-f)$

$$\begin{array}{rcccccccc}
x(t) = & p_R & +j & p_I & + & n_R & +j & n_I \\
 & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
X(f) = & P_R & +j & P_I & +j & N_I & + & N_R
\end{array}$$

$$x(t) = \quad x_P(t) \quad + \quad x_N(t)$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$X(f) = \operatorname{sinc}^2(f)\cos(2\pi f) \; - \; j\operatorname{sinc}^2(f)\operatorname{sen}(2\pi f)$$



d.II **Propiedad de derivación**

$$\begin{aligned}
\frac{dx}{dt} &= \frac{d}{dt}\left(\int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,df\right) \\
&= \int_{-\infty}^{\infty} X(f)\frac{d}{dt}\left(e^{j2\pi ft}\right)df \\
&= \int_{-\infty}^{\infty} j2\pi f X(f)e^{j2\pi ft}\,df \\
&= \mathcal{F}^{-1}\{j2\pi f X(f)\}
\end{aligned}$$

$$\frac{dx}{dt} \supset j2\pi f X(f)$$

# Ejercicio 1

**e.I** $U(f) = \mathcal{F}\{u\}(f) = ?$

$$x(t) = x_0(t) + \bar{x} \qquad \bar{x} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\,dt$$

$$u(t) = u_0(t) + \bar{u} \qquad \bar{u} = \lim_{T\to\infty} \frac{1}{2T}\int_{0}^{T} 1\,dt = \frac{1}{2}$$

$u'(t) = \delta(t)$ pero $u_0'(t) = \delta(t)$

$$j2\pi f \mathcal{F}\{u_0\}(f) = 1$$

$$j2\pi f \mathcal{F}\{u - \bar{u}\}(f) = 1$$

$$j2\pi f\,(U(f) - \bar{u}\delta(f)) = 1$$

$$U(f) = \frac{1}{j2\pi f} + \frac{\delta(f)}{2}$$

$$\frac{dx}{dt} \supset j2\pi\, f X(f)$$

$$u_0(t) = u(t) - \bar{u}$$

$$1 \supset \delta(f)$$

# Ejercicio 1

**e.II**

$$\mathrm{sgn}(t) = 2u(t) - 1$$

**e.III**

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau = \int_{-\infty}^{\infty} x(\tau)u(t-\tau)\, d\tau \; = \{x * u\}(t)$$

$$\begin{aligned} Y(f) &\supset X(f)U(f) \\ &= \frac{X(f)}{j2\pi\, f} + \frac{X(f)\delta(f)}{2} \\ \int_{-\infty}^{t} x(\tau)\, d\tau &\supset \frac{X(f)}{j2\pi\, f} + \frac{X(0)}{2}\delta(f) \end{aligned}$$

$$\{x * y\}(t) \supset X(f)Y(f)$$

$$u(t) \supset \frac{1}{2}\left(\delta(f) + \frac{1}{j\pi\, f}\right)$$

# Ejercicio 2

a.IV $x(t) = e^{-j\pi t}$

$$1 \supset \delta(f)$$

$$e^{-j2\pi\frac{1}{2}t}1 \supset \delta(f + \frac{1}{2}) \qquad x(t)e^{j2\pi f_0 t} \supset X(f - f_0)$$

a.I $x(t) = \wedge(t/8 - 3)$

$$\wedge(t) \supset \mathrm{sinc}^2(f)$$

$$g(\tau) = \wedge(\tau/8) \supset 8\,\mathrm{sinc}^2(8f) \qquad x(a\,t) \supset \frac{1}{|a|}X\left(\frac{f}{a}\right)$$

$$g(t - 24) \supset$$

$$\wedge\left(\frac{t-24}{8}\right) \supset 8\,\mathrm{sinc}^2(8f)e^{-j2\pi 24 f} \qquad x(t - t_0) \supset e^{-j2\pi f t_0}X(f)$$

# Ejercicio 2

a.III

$$x(t) = e^{-t^2/2-3t-1} = e^{-\frac{1}{2}\left(t^2+6t+2\right)}$$
$$= e^{-\frac{1}{2}(t+3)^2-7/2} = e^{-\frac{1}{2}(t+3)^2}e^{-7/2}$$

$$e^{-t^2/2\sigma^2} \supset \sqrt{2\pi\sigma^2}e^{-(\pi f)^2 2\sigma^2}$$
$$e^{-\frac{\tau^2}{2}} \supset \sqrt{2\pi}e^{-(\pi f)^2 2}$$
$$e^{-\frac{1}{2}(t+3)^2} \supset \sqrt{2\pi}e^{-2(\pi f)^2}e^{j2\pi 3f}$$
$$e^{-\frac{1}{2}(t+3)^2}e^{-7/2} \supset \sqrt{2\pi}e^{-2(\pi f)^2}e^{j2\pi 3f}e^{-7/2}$$

$$\sigma^2 = 1$$
$$x(t-t_0) \supset e^{-j2\pi f t_0}X(f)$$

# Ejercicio 2

a.IX $x(t) = \mathrm{sen}(\pi t) e^{-t/4} u(t)$

$$\mathrm{sen}(\pi t) \supset$$

$$\frac{e^{j\pi f} - e^{-j\pi f}}{2j} \supset \frac{1}{2j}\delta\left(f - \frac{1}{2}\right) - \frac{1}{2j}\delta\left(f + \frac{1}{2}\right) \qquad x(t)e^{j2\pi f_0 t} \supset X(f - f_0)$$

$$e^{-\alpha t}u(t) \supset \frac{1}{\alpha + j2\pi\, f} \qquad \longrightarrow \qquad e^{-t/4}u(t) \supset \frac{1}{\frac{1}{4} + j2\pi\, f}$$

$$x(t)y(t) \supset \{X * Y\}(f)$$

$$\mathrm{sen}(\pi t)e^{-\frac{t}{4}}u(t) \supset \left\{\left(\frac{1}{2j}\delta\left(\cdot - \frac{1}{2}\right) - \frac{1}{2j}\delta\left(\cdot + \frac{1}{2}\right)\right) * \frac{1}{\frac{1}{4} + j2\pi\,\cdot}\right\}(f)$$

$$\mathrm{sen}(\pi t)e^{-\frac{t}{4}}u(t) \supset \frac{1/2j}{\frac{1}{4} + j2\pi(f - \frac{1}{2})} - \frac{1/2j}{\frac{1}{4} + j2\pi(f + \frac{1}{2})}$$

# Ejercicio 2

a.XI $x(t) = \left\{ e^{-(t-2)} u(t-2) * \mathrm{sinc}(t/2) \right\}(t)$

$$e^{-\alpha t} u(t) \supset \frac{1}{\alpha + j2\pi f} \quad \longrightarrow \quad e^{-t} u(t) \supset \frac{1}{1 + j2\pi}$$

$$e^{-(t-2)} u(t-2) \supset \frac{e^{-j2\pi 2f}}{1 + j2\pi f} \qquad x(t - t_0) \supset e^{-j2\pi f t_0} X(f)$$

$$\sqcap(t) \supset \mathrm{sinc}(f)$$

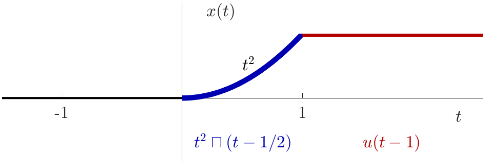
$$\mathrm{sinc}(t) \supset \sqcap(f) \qquad X(-t) \supset x(f)$$

$$\mathrm{sinc}(t/2) \supset 2 \sqcap(2f) \qquad x(a\,t) \supset \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

$$\{x * y\}(t) \supset X(f) Y(f)$$

$$\left\{ e^{-(t-2)} u(t-2) * \mathrm{sinc}(t/2) \right\}(t) \supset \frac{e^{-j2\pi 2f}}{1 + j2\pi f} 2 \sqcap(2f)$$

## a.XII




$$x(t) = t^2 \sqcap (t - 1/2) + u(t-1)$$

$$\int t^2 e^{\alpha t}\, dt = \frac{e^{\alpha t}}{\alpha}\left[t^2 - \frac{2t}{\alpha} + \frac{2}{\alpha^2}\right]$$

# Ejercicio 2

b.I $X(f) = \wedge(2f) - jf \sqcap(f)$

$$\wedge(t) \supset \text{sinc}^2(f)$$

$$\text{sinc}^2(t) \supset \wedge(f) \qquad X(-t) \supset x(f)$$

$$\frac{1}{2}\,\text{sinc}^2\left(\frac{t}{2}\right) \supset \frac{1}{2} 2 \wedge(2f) \qquad x(a\,t) \supset \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

$$\text{sinc}(t) \supset \sqcap(f)$$

$$\frac{d\{\text{sinc}(t)\}}{dt} \supset j2\pi f \sqcap(f) \qquad \frac{dx}{dt} \supset j2\pi f X(f)$$

$$-\frac{1}{2\pi}\frac{d\{\text{sinc}(t)\}}{dt} \supset -jf \sqcap(f)$$

$$\frac{1}{2}\,\text{sinc}^2\left(\frac{t}{2}\right) - \frac{1}{2\pi}\frac{d\{\text{sinc}(t)\}}{dt} \supset \wedge(2f) - jf \sqcap(f)$$

# Ejercicio 2

b.IV $X(f) = \text{sen}\left(4\pi f - \frac{\pi}{6}\right)$

$$\text{sen}(2\pi t) \supset \frac{1}{2j}\delta\left(f - \frac{1}{2}\right) - \frac{1}{2j}\delta\left(f + \frac{1}{2}\right)$$

$$X(-t) \supset x(f)$$

$$\frac{1}{2j}\delta\left(-t - \frac{1}{2}\right) - \frac{1}{2j}\delta\left(-t + \frac{1}{2}\right) \supset \text{sen}(2\pi f)$$

$$x(t)e^{j2\pi f_0 t} \supset X(f - f_0)$$

$$\frac{e^{j2\pi^2 t/6}}{2j}\delta\left(t + \frac{1}{2}\right) - \frac{e^{j2\pi^2 t/6}}{2j}\delta\left(t - \frac{1}{2}\right) \supset \text{sen}\left(2\pi f - \frac{\pi}{6}\right)$$

$$x(a\,t) \supset \frac{1}{|a|}X\left(\frac{f}{a}\right)$$

$$\frac{e^{j2\pi^2 t/12}}{4j}\delta\left(\frac{t+1}{2}\right) - \frac{e^{j2\pi^2 t/12}}{4j}\delta\left(\frac{t-1}{2}\right) \supset \text{sen}\left(2\pi(2f) - \frac{\pi}{6}\right)$$