SEÑALES Y SISTEMAS

Práctica 4
Ejercicios 3, 4 y 5

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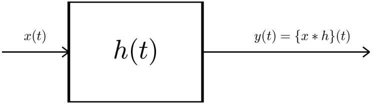
Ejercicio 3

Respuesta en frecuencia de SLIT



$$\{x * y\}(t) \supset X(f)Y(f)$$

a. $x(t) = Ae^{j(2\pi f_0 t + \theta)}$

$$y(t) = \int_{\infty}^{\infty} Ae^{j(2\pi f_0 (t-\tau) + \theta)} h(\tau)\, d\tau$$

$$= Ae^{j(2\pi f_0 t + \theta)} \underbrace{\int_{\infty}^{\infty} h(\tau) e^{-j2\pi f_0 \tau}\, d\tau}_{H(f_0)}$$

$$= AH(f_0)e^{j(2\pi f_0 t + \theta)} = A|H(f_0)|e^{j(2\pi f_0 t + \theta) + j\angle H(f_0)}$$

Ejercicio 3

b.

$$x(t) = Ae^{j2\pi f_0 t}e^{j\theta} \supset Ae^{j\theta}\delta(f - f_0)$$

$$h(t) \supset H(f)$$

$$Y(f) = X(f)H(f) = Ae^{j\theta}\delta(f - f_0)H(f)$$

$$= Ae^{j\theta}\delta(f - f_0)H(f_0)$$

$$AH(f_0)e^{j2\pi f_0 t}e^{j\theta} \supset Ae^{j\theta}H(f_0)\delta(f - f_0)$$

$$y(t) = AH(f_0)e^{j(2\pi f_0 t + \theta)}$$

g.

$$y'(t) + 3y(t) = x(t) \longrightarrow j2\pi fY(f) + 3Y(f) = X(f)$$

$$H(f) = \frac{Y(f)}{H(f)} = \frac{1}{3 + j2\pi f}$$

$$h(t) = e^{-3t}u(t)$$

Ejercicio 3

g.l d $$x(t) = \text{sen}(6\pi t)$$

$$\text{sen}(6\pi t) = \frac{1}{2j}\left(e^{j2\pi 3t} - e^{-j2\pi 3t}\right) \supset \frac{1}{2j}\left(\delta(f-3) - \delta(f+3)\right)$$

$$\begin{aligned} y(t) &= \frac{1}{2j}\left(H(3)e^{j2\pi 3t} - H(-3)e^{-j2\pi 3t}\right) \\ &= \frac{1}{2j}\left(|H(3)|e^{j2\pi 3t + j\angle H(3)} - |H(-3)|e^{-j2\pi 3t + j\angle H(-3)}\right) \end{aligned}$$

$$h(t) \in \mathbb{R} \longrightarrow |H(3)| = |H(-3)| \text{ y } \angle H(3) = -\angle H(-3)$$

$$\begin{aligned} y(t) &= \frac{1}{2j}\left(|H(3)|e^{j2\pi 3t + j\angle H(3)} - |H(3)|e^{-j2\pi 3t - j\angle H(3)}\right) \\ &= |H(3)|\frac{1}{2j}\left(e^{j2\pi 3t + j\angle H(3)} - e^{-j2\pi 3t - j\angle H(3)}\right) \\ &= |H(3)|\,\text{sen}(6\pi t + \angle H(3)) \end{aligned}$$

Ejercicio 3

$$y(t) = \left|\frac{1}{3+j6\pi}\right| \operatorname{sen}\left(6\pi t + \angle\frac{1}{3+j6\pi}\right)$$
$$= \frac{1}{\sqrt{9+36\pi^2}} \operatorname{sen}\left(6\pi t - \operatorname{arc\,tg}(2\pi)\right)$$

g.V $x(t) = \sqcap(t) \supset X(f) = \operatorname{sinc}(f)$

$$Y(f) = X(f)H(f) = \frac{\operatorname{sinc}(f)}{3+j2\pi f}$$
$$= \frac{1}{f(3+j2\pi f)}\frac{e^{j\pi f}}{2j\pi} - \frac{1}{2j\pi f(3+j2\pi f)}\frac{e^{-j\pi f}}{2j\pi}$$

$$\frac{1}{f(3+j2\pi f)} = \frac{A}{f} + \frac{B}{3+j2\pi f} = \frac{3A + j2\pi f A + Bf}{f(3+j2\pi f)}$$

Ejercicio 3

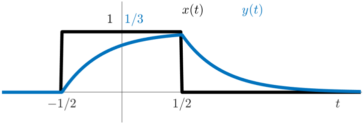
$$3A = 1 \qquad A = 1/3$$

$$B + j2\pi A = 0 \qquad B = -j2\pi/3$$

$$Y(f) = \frac{e^{j2\pi f \frac{1}{2}}}{2j\pi}\left[\frac{1/3}{f} - \frac{j2\pi/3}{3+j2\pi f}\right] - \frac{e^{-j2\pi f \frac{1}{2}}}{2j\pi}\left[\frac{1/3}{f} - \frac{j2\pi/3}{3+j2\pi f}\right]$$

$$j\frac{\pi}{3}\operatorname{sgn}(t) \supset \frac{1/3}{f}$$

$$-j\frac{2\pi}{3}e^{-3t}u(t) \supset -\frac{j2\pi/3}{3+j2\pi f}$$

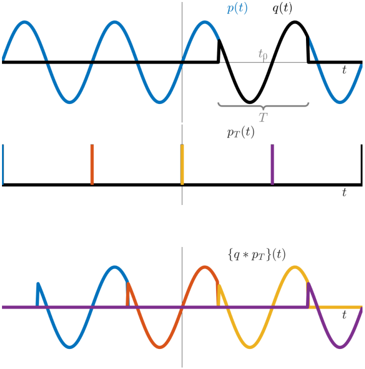


$$y(t) = \frac{1}{6}\operatorname{sgn}\left(t+\frac{1}{2}\right) - \frac{1}{3}e^{-3(t+1/2)}u(t+1/2) -$$

$$-\frac{1}{6}\operatorname{sgn}\left(t-\frac{1}{2}\right) + \frac{1}{3}e^{-3(t-1/2)}u(t-1/2)$$

Ejercicio 4

a. $p(t)$ señal periódica de período T



$$q(t) = p(t) \sqcap \left(\frac{t - t_0}{T} \right)$$

$$p_T(t) = \frac{1}{T} \text{Ш} \left(\frac{t}{T} \right)$$

$$p(t) = \{q * p_T\}(t)$$

$$\text{Ш}(t) \supset \text{Ш}(f)$$

$$\text{Ш} \left(\frac{t}{T} \right) \supset T \, \text{Ш}(fT)$$

$$\frac{1}{T} \text{Ш} \left(\frac{t}{T} \right) \supset \text{Ш}(fT) = \sum_{k=-\infty}^{\infty} \delta\left(fT - k\right) = \sum_{k=-\infty}^{\infty} \frac{1}{T} \delta\left(f - \frac{k}{T}\right)$$

$$P(f) = Q(f) \sum_{k=-\infty}^{\infty} \frac{1}{T}\delta\left(f - \frac{k}{T}\right) = \sum_{k=-\infty}^{\infty} \frac{1}{T} Q(f)\delta\left(f - \frac{k}{T}\right)$$

$$= \sum_{k=-\infty}^{\infty} \frac{1}{T} Q\left(\frac{k}{T}\right)\delta\left(f - \frac{k}{T}\right)$$

b.

$$c_k = \frac{1}{T}\int_T p(t) e^{-j2\pi kt/T}\, dt = \frac{1}{T}\int_{-\infty}^{\infty} p(t) \sqcap \left(\frac{t-t_0}{T}\right) e^{-j2\pi kt/T}\, dt$$

$$= \frac{1}{T}\int_{-\infty}^{\infty} q(t) e^{-j2\pi \frac{k}{T} t}\, dt = \frac{1}{T} Q\left(\frac{k}{T}\right)$$

c.

$$P(f) = \sum_{k=-\infty}^{\infty} c_k \delta\left(f - \frac{k}{T}\right)$$

Ejercicio 4

e. $p(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi\frac{k}{T}t}$

$$P_p = \frac{1}{T}\int_T p(t)p^*(t)\,dt = \frac{1}{T}\int_T \sum_{k=-\infty}^{\infty} c_k e^{j2\pi\frac{k}{T}t} \sum_{l=-\infty}^{\infty} c_l^* e^{-j2\pi\frac{l}{T}t}\,dt$$

$$= \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} c_k c_l^* \underbrace{\frac{1}{T}\int_T e^{j2\pi\frac{k}{T}t} e^{-j2\pi\frac{l}{T}t}\,dt}_{= \begin{cases} 1 & k=l \\ \frac{e^{j\pi(k-l)} - e^{-j\pi(k-l)}}{j2\pi(k-l)} = \frac{\operatorname{sen}(\pi(k-l))}{\pi(k-l)} = 0 & k \neq l \end{cases}}$$

$$= \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} c_k c_l^* \delta[k-l] = \sum_{k=-\infty}^{\infty} c_k c_k^* = \sum_{k=-\infty}^{\infty} |c_k|^2$$

$$c_0 = \frac{1}{T}\int_T p(t) e^{-j2\pi 0 t/T}\,dt = \bar{p}$$

Ejercicio 4

f. $p(t) = e^{-j10\pi t}, \quad T = 1/5$

$$
\begin{aligned}
c_k &= \frac{1}{T}\int_T p(t) e^{-j2\pi kt/T}\,dt = 5\int_0^{1/5} e^{-j10\pi t} e^{-j10\pi kt}\,dt \\
&= 5\int_0^{1/5} e^{-j10\pi(k+1)t}\,dt = \begin{cases} 1 & k=-1 \\ \dfrac{e^{-j2\pi(k+1)}-1}{-j10\pi(k+1)} = 0 & k \neq -1 \end{cases} \\
&= \delta[k+1]
\end{aligned}
$$

otra forma

$$
q(t) = e^{-j10\pi t} \sqcap (5t) \supset \frac{1}{5}\,\mathrm{sinc}\left(\frac{f+5}{5}\right) = Q(f)
$$

$$
c_k = \frac{1}{T} Q\left(\frac{k}{T}\right) = 5Q(5k) = \mathrm{sinc}(k+1) = \delta[k+1]
$$

Ejercicio 5

a. $x(t)$ periódica $\longrightarrow y(t) = \{x * h\}(t)$ periódica?

$$x(t) = \sum_{k=-\infty}^{\infty} c_x[k] e^{j2\pi \frac{k}{T} t}$$

SLIT $h(t) \supset H(f)$

$$e^{j2\pi \frac{k}{T} t} \longrightarrow e^{j2\pi \frac{k}{T} t} H\left(\frac{k}{T}\right)$$

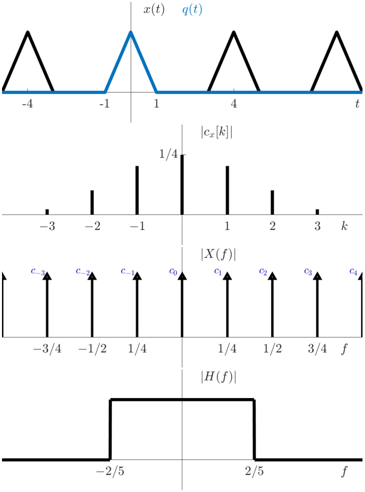
$$y(t) = \sum_{k=-\infty}^{\infty} c_x[k] e^{j2\pi \frac{k}{T} t} H\left(\frac{k}{T}\right)$$

$$= \sum_{k=-\infty}^{\infty} c_x[k] H\left(\frac{k}{T}\right) e^{j2\pi \frac{k}{T} t}$$

$$c_y[k] = c_x[k] H\left(\frac{k}{T}\right)$$

b.

$$x(t) = \{\wedge * p_4\}(t)$$
$$q(t) = \wedge(t) \supset Q(f) = \text{sinc}^2(f)$$



SF:

$$c_x[k] = \frac{1}{T} Q\left(\frac{k}{T}\right) = \frac{1}{4}\,\text{sinc}^2\left(\frac{k}{4}\right)$$

TF:

$$X(f) = \sum_{k=-\infty}^{\infty} \frac{1}{4}\,\text{sinc}^2\left(\frac{k}{4}\right)\delta\left(f - \frac{k}{T}\right)$$

Sistema:

$$h(t) = \frac{4}{5}\,\text{sinc}\left(\frac{4}{5}t\right) \supset H(f) = \sqcap\left(\frac{5f}{4}\right)$$

TF salida:

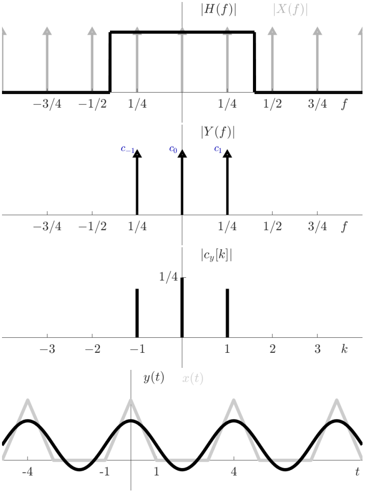
$Y(f) = X(f)H(f)$

SF salida:

$$c_y[k] = c_x[k]H\left(\frac{k}{T}\right)$$
$$= \frac{1}{4}\,\mathrm{sinc}^2\left(\frac{k}{4}\right)\sqcap\left(\frac{f}{5}\right)$$
$$= \begin{cases} 1/4 & k = 0 \\ \frac{1}{4}\,\mathrm{sinc}^2\left(\frac{1}{4}\right) & |k| = 1 \\ 0 & |k| > 1 \end{cases}$$

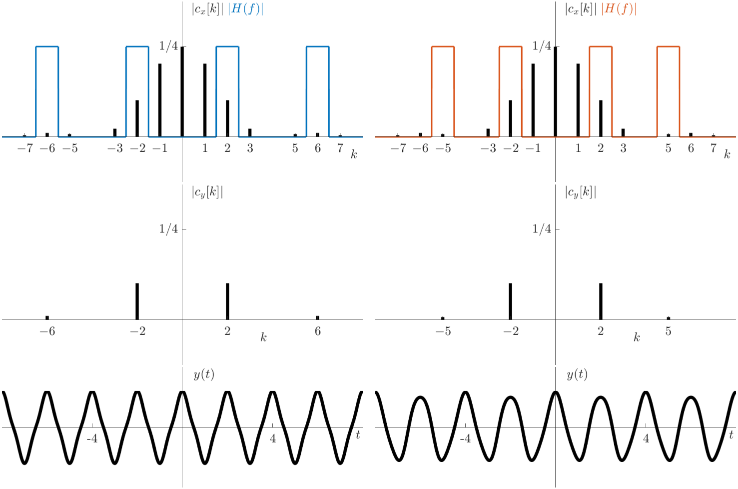
Salida:

$y(t) = \frac{1}{4} + 2c_y[1]\cos(\pi f/2)$



Ejercicio 5

Otros Filtros



frecuencia fundamental $= 1/2$

frecuencia fundamental $= 1/4$

Ejercicio 5

c. $x(t) = \cos(2\pi t)$

- $$q(t) = \cos(2\pi t) \sqcap (t)$$
  $$Q(f) = \left\{ \frac{\delta(f+1) + \delta(f-1)}{2} * \operatorname{sinc}(f) \right\}$$
  $$Q(f) = \frac{\operatorname{sinc}(f+1) + \operatorname{sinc}(f-1)}{2}$$
  $$c_k = Q\left(k\right) = \frac{\operatorname{sinc}(k+1) + \operatorname{sinc}(k-1)}{2} = \frac{1}{2}\delta[k+1] + \frac{1}{2}\delta[k-1]$$

- $$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi \frac{k}{T} t}$$
  $$x(t) = \cos(2\pi t) = \underbrace{\frac{1}{2}}_{c_1} e^{\overbrace{j2\pi t}^{2\pi(1)t/1}} + \underbrace{\frac{1}{2}}_{c_{-1}} e^{\overbrace{-j2\pi t}^{2\pi(-1)t/1}}$$

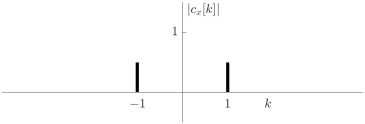
Ejercicio 5

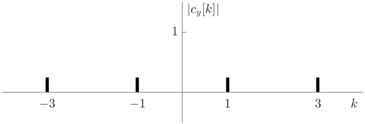
$$y(t) = \cos(\pi t)x(t) = \cos(\pi t)\cos(2\pi t) = \frac{1}{2}\underbrace{\cos(\pi t)}_{T_1=2} + \frac{1}{2}\underbrace{\cos(3\pi t)}_{T_2=2/3}$$

$$\frac{T_1}{T_2} = \frac{2}{2/3} = 3 \qquad T_1 = 3T_2 = T = 2$$

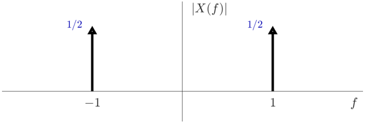
$$y(t) = \frac{1}{4}e^{j\pi t} + \frac{1}{4}e^{-j\pi t} + \frac{1}{4}e^{j3\pi t} + \frac{1}{4}e^{-j3\pi t}$$

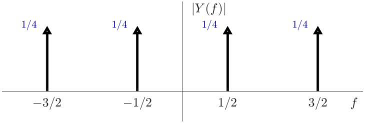
$$= \underbrace{\frac{1}{4}}_{c_1} e^{\overbrace{j2\pi(1)\frac{t}{2}}^{k=1}} + \underbrace{\frac{1}{4}}_{c_{-1}} e^{\overbrace{j2\pi(-1)\frac{t}{2}}^{k=-1}} + \underbrace{\frac{1}{4}}_{c_3} e^{\overbrace{j2\pi(3)\frac{t}{2}}^{k=3}} + \underbrace{\frac{1}{4}}_{c_{-3}} e^{\overbrace{j\pi(-3)\frac{t}{2}}^{k=-3}}$$

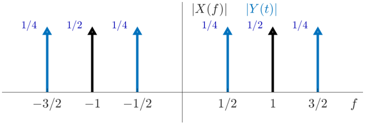




$f = k/1$ $\qquad$ $f = k/2$







Nuevas Frecuencias! $\longrightarrow$ NO LINEAL